OF MICROBUBBLES IN TURBULENT FLOW

V. G. Gavrilenko and A. I. Mart'yanov

Theoretical and experimental research [1-3] has shown that the gas bubble-size distribution function may vary considerably both in still water and in turbulent flow. These changes are frequently unrelated to the creation or collapse of bubbles which occur under cavitation conditions, but arise from the relatively slow processes of solution or growth of gas nuclei. A rigorous theoretical treatment of bubble growth processes, particularly in turbulent flow, encounters a whole series of difficulties arising from the uncertainty of certain important physical parameters (amount and composition of dissolved gas in the liquid surrounding the bubbles, the ratio of bubble sizes to the internal scale of turbulence, the purity of the liquid, etc.). The experimental determination of the rate of growth of a bubble also encounters technical difficulties. It is very difficult to trace a single rapidly moving bubble by photographic recording devices, and the behavior of a single fixed bubble in the stream may be very different from the behavior of a free gas nucleus. We examine the possibility of assessing the rate of growth of free bubbles in a turbulent submerged water jet by measuring the distribution function of gas nuclei and its evolution along the axis of the jet.

Assuming that there is no instantaneous creation or collapse of bubbles in the part of the jet under investigation, and that the size distribution function changes only as a result of growth (solution), the equation of continuity for the concentration of gas nuclei in the reference system moving with the average flow velocity can be written in the form

$$
\begin{equation*}
\partial n / \partial t+\gamma \partial n / \partial R+n \partial \gamma / \partial R=0 \tag{1}
\end{equation*}
$$

where n is the number of bubbles per unit range of radii in a layer between two neighboring cross sections of the jet, $\gamma=\partial \mathrm{R} / \partial \mathrm{t}$, and R is the radius of a bubble. The validity of the above assumption that there are no instantaneous processes was confirmed experimentally. If the jet does not emerge from the cavitating nozzle but in the region where the measurements are made, and the flow remains the same as for the discharge from the cavitating nozzle, then the number of gas nuclei is very small and in practice cannot be recorded by the apparatus used. And in the discharge from the cavitating nozzle the total number of bubbles in the range of radii being monitored in the layer varies slowly along the axis of the jet; $i_{.} e_{\text {. }}$ it can be assumed that the gas nuclei which take part in the processes of creation and collapse constitute a very small fraction of the total number of gas nuclei. Also in the portion of the jet being investigated there is no production of acoustic noise which is characteristic of the processes of creation and collapse of bubbles and is clearly recorded in the immediate vicinity of the cavitating nozzle.

Equation (1) is easily solved if $\partial \gamma / \partial \mathrm{R} \rightarrow 0$ for $\gamma \neq 0$, and both experiment and theory indicate that this case is realized for certain values of $R$. In practice it is more convenient to calculate $\gamma$ by operating with the number of bubbles in a certain finite range of radii $\Delta R$ rather than with concentrations. To find $\gamma$ it is necessary to know how the total number of bubles in each range of radii over the whole range of radii monitored varies along the axis of the jet. With this in mind we measured the attenuation of a narrow ultrasonic beam propagating across the jet at various distances $x$ from the lip of the cavitating nozzle. The bubblesize distribution function in various parts of the jet was found by using these data, theoretical caluclations in [4], and the method of processing in [3].

Within the framework of the model chosen, the change in the total number of bubbles in a subgroup can be written as

$$
\begin{equation*}
d N_{i}=q_{i-1 \rightarrow i}-q_{i \rightarrow i+1}, \tag{2}
\end{equation*}
$$

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where $N_{i}$ is the total number of bubbles in the layer in the $i$-th subgroup in the range of radii $\Delta R ; q_{i-1} \rightarrow i$, number of bubbles entering group i from the adjacent subgroup $i-1$ with smaller radii in the portion of the jet of length $d x ; q_{i \rightarrow i+1}$, number of bubbles leaving subgroup $i$ and entering the adjacent subgroup $i+1$ with larger radii.

Assuming that the change in the bubble-size distribution density within a subgroup is small, we can write

$$
\begin{equation*}
q_{i-1 \rightarrow i}=\frac{N_{i-1}+N_{i}}{2} \frac{\gamma_{j}^{\prime} d x}{\Delta R}, \quad q_{i \rightarrow i+1}=\frac{N_{i}+N_{i+1}}{2} \frac{\gamma_{j}^{\prime} d x}{\Delta R} . \tag{3}
\end{equation*}
$$

where $\gamma_{j}^{\prime}=(\partial \mathrm{R} / \partial \mathrm{x})_{R_{j}}$ (the $\mathrm{R}_{\mathrm{j}}$ are the boundary values of the radii of bubbles of the two adjacent groups $\mathrm{i}-1$ and i); $\gamma_{\mathbf{j + 1}}^{\prime}=(\partial R / \partial x)_{\mathbf{R}_{\mathbf{j}+1}}$ (the $\mathrm{R}_{\mathrm{j}+1}$ are the boundary values of the radii of bubbles of the two adjacent groups $i$ and $i+1$ ). After substituting (3) into (2) we obtain

$$
\begin{equation*}
\frac{d N_{i}}{d x} 2 \Delta R=\left(N_{i-1}+N_{i}\right) \gamma_{j}^{\prime}-\left(N_{i}+N_{i+1}\right) \gamma_{j+1}^{\prime} \tag{4}
\end{equation*}
$$

By letting $\Delta R \rightarrow 0$ and replacing $d x$ by Vdt, where $V$ is the average velocity of the jet, Eq. (4) goes over into (1).

Assuming that

$$
\begin{equation*}
\left|\gamma_{j+1}^{\prime}-\gamma_{j}^{\prime}\right| \ll \gamma_{j}^{\prime} \tag{5}
\end{equation*}
$$

We can write $\gamma_{j}^{\prime}$

$$
\begin{equation*}
\gamma_{j}^{\prime}=\frac{d N_{i}}{d x} \frac{2 \Delta R}{N_{i-1}-N_{i+1}} \tag{6}
\end{equation*}
$$

By calculating $\gamma_{j}^{\prime}$ for various $i$, i.e., for various radii of gas nuclei, it is possible to find a range of values of radii (if one exists) for which inequality (5) is satisfied. With the value of $\gamma_{\mathbf{j}}^{\prime}$ calculated in the range of applicability of Eq. (6), Eq. (4) can be used to find the values of $\gamma_{j \pm 1}^{\prime}$, i.e., the rate of growth of bubbles at the boundary of the adjacent interval.

An experiment was performed in a hydroacoustic basin to investigate the evolution of the bubble distribution function in a circular pipe and a submerged jet as a function of the distance from the lip of the nozzle. The range of radii monitored included values from $2.25 \cdot 10^{-4} \mathrm{~cm}$ to $13 \cdot 10^{-4} \mathrm{~cm}$. The whole range was divided into 22 intervals, each of width $\Delta \mathrm{R}=0.5 \cdot 10^{-4} \mathrm{~cm}$. The distribution functions found were used to construct the relations $\mathrm{N}_{\mathrm{i}}(\mathrm{x})$ for all groups. Figure 1 shows a family of curves for several values of $\mathrm{R}_{\mathrm{i}}$, the average radius of a bubble in a subgroup in the submerged jet: 1) $R=3.5 \cdot 10^{-4} \mathrm{~cm}$; 2) $R=4 \cdot 10^{-4} \mathrm{~cm}$; 3) $R=5 \cdot 10^{-4} \mathrm{~cm}$; 4) $R=6.5 \cdot 10^{-4} \mathrm{~cm}$; 5) $R=8 \cdot 10^{-4} \mathrm{~cm}$. ( $\mathrm{N}_{i}$ is the total number of bubbles in the cross section of the jet in a layer 0.51 mm thick. The volume of a layer of this thickness at the lip of the nozzle is $\sim 1 \mathrm{~cm}^{3}$ 。)

Values of $\gamma_{j}^{\prime}$ at various distances from the lip of the nozzle were calculated from EqS. (5) and (4) and multiplied by the speed of the jet to obtain the values of $\gamma_{j}$. The values found for $\gamma(R, X)$ are listed in Table 1.

Investigation of the distribution function of gas nuclei in a circular pipe showed that at least at distances from 10 to 120 cm from the lip of the cavitating nozzle the distribution function changed so little that the rate of growth of bubbles could be called zero.
(In investigating flow in a pipe $\sim 4.5 \mathrm{~cm}$ in diameter the distribution function was taken directly at the pipe exit, and the cavitating nozzle, which fit closely inside the pipe, was moved along it.)

From an analysis of the experimental results obtained certain assumptions can be made about the fundamental causes of bubble growth in turbulent flow. In the discharge of a liquid from a cavitating nozzle into a pipe in running water it takes $\sim 10^{-2} \mathrm{sec}$ to establish a certain equilibrium state between the free gas in bubbles and that dissolved in the water. If the jet discharges into a infinite volume, undisturbed masses of water are entrained into the flow. Upon entering a region of lower pressure these masses of water become
resaturated by dissolved gas, and this leads to an increase in the diffusion of gas into bubbles causing them to grow. Experiment shows that the rate of growth of relatively large bubbles with radii $\sim(7-9) \cdot 10^{-4} \mathrm{~cm}$ is proportional to the magnitude of the velocity head.

The rate of growth of smaller bubbles varies somewhat differently with the distance from the lip of the nozzle, and beginning at certain distances becomes negative. This clearly results from the fact that the dynamics of bubbles depends strongly on the ratio of their sizes to that of the internal scale of turbulence. In this connection one rule observed in the experiment can be noted. If the internal scale defined [5] as $l=$ $L / \operatorname{Re}^{3 / 4}$, where $L$ is the transverse dimension of the stream and $R e$ is the Reynolds number, is compared with the diameter of the bubbles, the number of bubbles in the $i$-th subgroup is maximum at distances from the lip of the nozzle where $l \approx 2 R_{i}$ (the values of $l$ for the stream are given along the lower axis of $F i g .1$ ).

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